Optimization of energy extraction for closed shallow geothermal systems using linear programming

Michael de Paly a,∗, Jozsef Hecht-Méndez b, Markus Beck a, Philipp Blum c, Andreas Zell a, Peter Bayer d

a University of Tübingen, Wilhelm-Schickard-Institute for Computer Science (WSI), Sand 1, 72076 Tübingen, Germany
b University of Tübingen, Center for Applied Geoscience (ZAG), Sigwartstraße 10, 72076 Tübingen, Germany
c Karlsruhe Institute of Technology (KIT), Institute for Applied Geosciences (AGW), Kaiserstraße 12, 76131 Karlsruhe, Germany
d ETH Zürich, Engineering Geology, Sonneggstrasse 5, 8092 Zürich, Switzerland

A R T I C L E   I N F O

Article history:
Received 21 January 2011
Accepted 19 March 2012
Available online 13 April 2012

Keywords:
Shallow geothermal systems
Sustainability
Optimization
Linear programming

A B S T R A C T

The objective of the study is to optimize the performance and thereby to mitigate the environmental impact of ground source heat pump (GSHP) systems using multiple borehole heat exchangers (BHEs) by including variable energy loads. Hence, an optimization procedure is developed that is able to predict temperature distributions in the subsurface. Optimized BHE fields are able to keep the maximum temperature change in the subsurface about 18% lower than BHE fields which feature equal flow rates for all BHEs. The long-term temperature anomaly can be mitigated and the possibility of extracting a higher amount of energy, while keeping temperature thresholds or environmental constraints, arises.

© 2012 Elsevier Ltd. All rights reserved.

1. Introduction

Shallow geothermal technologies that directly use the heat stored in the subsurface are counted among the most sustainable choices for space heating (Rybach and Mongillo, 2006; Saner et al., 2010). Depending on the supplied primary energy for the heat pumps and the efficiency of installation CO2 can be avoided or even reduced (Nagano et al., 2006; Blum et al., 2010). In areas of normal or low geothermal gradient, heat pumps can be applied to extract the energy from depths down to about 400 m. Most often, closed or ground-coupled systems such as ground source heat pump (GSHP) systems are used (Lund et al., 2010). Depending on the heating demand, one or several vertical borehole exchangers (BHEs) are typically installed in the ground. The system is operated by circulating a heat carrier fluid in the BHEs, which exchanges heat with the ground and feeds an indoor heat pump. Even if shallow subsurface energy resources are enormous, local extraction causes temperature anomalies (Philippe et al., 2009; Hecht-Méndez et al., 2010). Natural heat conduction automatically balances these anomalies (Signorelli et al., 2004). However, replenishment needs time and hence the ground temperature should not be altered significantly. Ideally GSHP systems are run in seasonal mode, so that summer break is available for progressive replenishment of the energy deficit (Rybach and Eugster, 2010).

In order to ensure a lifetime of decades, it is crucial to configure the BHEs appropriately. This means that during the heating period the commonly rather low temperature of the ground should only be decreased by a few degrees. This condition is also necessary to maximize the performance of the heat pump, which is most efficient when the temperature increment between heat source and receiver is diminutive (e.g., Esen et al., 2008). In addition to technical issues environmental concerns might play a role such as the leakage of anti-freeze liquids used in BHEs (Klotzbücher et al., 2007; Saner et al., 2010) or the impact on the groundwater ecosystem by temperature changes (Brielmann et al., 2009, 2011). Sometimes artificial temperature changes are also regulated (Haehnléin et al., 2010). For example, in the state capital of Stuttgart in South Germany, the impact on the groundwater temperature in a distance of 50 m from a ground heat exchanger should be ≤2 K and for larger GSHP systems (>10 BHEs) a temperature monitoring is often required by the environmental authorities (Amt für Umweltschutz, 2005). Hence, it is desirable to mitigate the thermal impact of such systems on the subsurface and groundwater.

Appropriate configuration of single and multiple BHEs usually follows standardized recipes and apposite planning software such as Earth Energy Designer (EED), Transient System Simulation Program with the Duct Ground Storage model (TRNSYS-DST) and other programs are available (Schmidt and Hellström, 2005; Nagano et al., 2006). For technical or legal reasons, BHEs commonly are not implemented deeper than a few hundreds of meters (Haehnléin et al., 2010). If a single BHE is not sufficient to supply the energy required for a specific case, BHE fields are constructed, and the total BHE...
length is oriented at the total energy demand. Depending on the specific case and available space, simple geometric arrangements such as square or longitudinal lattices are commonly selected (Fujii et al., 2005; Katsura et al., 2008, 2009). Usually, a distance of a few meters is considered sufficient to minimize mutual interference of adjacent BHEs, for example, from 3 m in China and up to 10 m in Germany (Kavanaugh and Rafferty, 1997; Signorelli et al., 2004; Haehnlein et al., 2010). BHE fields cause regional temperature anomalies, and projected to the surface these large scale temperature anomalies are composed of an array of tens to hundreds of concentric or elliptic temperature plumes that evolve around the BHEs. The larger the BHE field is the less effective the natural lateral conductive heat supply is, so that often the temperature within the field successively declines (Signorelli et al., 2004; Lazzari et al., 2010). Seasonal energy use in heating dominated operation is then reflected by periodic temperature variations that have a long-term decreasing trend (Diao et al., 2010). Accordingly, for a certain expected lifetime of a BHE field the final minimal ground temperature represents a crucial planning criterion. Thus, the aim is to keep the temperature decrease in the subsurface minimal while maximizing the technological performance of the BHEs.

In the present study, we identify thus far unconsidered degrees of freedom when planning BHE fields in conduction dominated media. In order to improve the concerted performance of multiple adjacent BHEs, the task is formulated as a mathematical optimization problem that is based on simulated superimposed BHE fields. Special attention is set on the heat transfer rates between the underground and the multiple BHEs that supply seasonal energy demands. Temperature changes in the underground exerted by multiple adjacent BHEs that operate with variable energy loads in heating mode are simulated. The temperature distribution can be predicted by numerical modeling (e.g., Bauer et al., 2011) or by spatial superposition of the infinite line source analytical solution (Carslaw and Jaeger, 1959). The time-dependent BHE-specific workloads are subdivided into step pulses and also superimposed (temporal superposition). An ideal energy extraction strategy is sought that takes into account the temperature changes in the underground and the time dependent heating demand. The optimal strategy supplies an effective input temperature to the heat pump and, therefore, is expected to improve the overall technical efficiency of the system compared to standard practice. Furthermore, the ideal solution should also guarantee lowest environmental impacts in the underground, i.e., aquifer temperatures should not be changed according to a pre-defined criterion or defined temperature limits and thresholds (Haehnlein et al., 2010).

Similar to previous work (e.g., Li et al., 2006, 2009; Katsura et al., 2008; Lazzari et al., 2010), we focus on optimization of BHE workloads (i.e., heat extraction rates). For example, Gao et al. (2010) present an operational strategy based on intermittent control of two BHEs; similarly, Cui et al. (2001) use discontinuous loads for the long-term operation of BHEs. While BHE specific loads can easily be simulated with the line source model, they do not exactly represent the real conditions in practice. In order to achieve a predefined load by operating BHEs, a control system that could, for example, adjust the flow rates of the heat carrier fluid would be necessary. Therefore, it is more common in practical applications to use equal flow rates for all BHEs, and by this, to indirectly obtain apposite BHE-specific energy extraction rates. Local low-temperature anomalies in the BHE field automatically lead to a smaller local energy extraction. Accordingly, in the well established BHE designing tools EED (Hellström and Sanner, 1997), and GLHEPRO (Spitler, 2000) the volumetric flow rate per borehole is uniformly distributed by the number of BHEs. A constant volumetric flow rate of the heat carrier can also be found in analytical and numerical simulation studies or performance investigations of multiple BHEs (Urchueguía et al., 2008; Katsura et al., 2009).

In the following, we will exploit the attractive properties from the analytical line source equation and, as a surrogate for flow rates, optimize BHE workloads in the field. The optimized solutions obtained by this proxy will be compared to standard practice, operating at equal flows and loads. The question is, even if flow regulation is not implicitly addressed in the mathematical formulation of the optimization problem, can the identified degrees of freedom be used to further improve BHE field design? To answer this question, the optimized results are compared to a BHE field implementation, in which a separate regulation of BHEs is not considered. Simulations by the Superposition Borehole Model (SBM, Eskilson, 1986; Palud et al., 1996), which is able to compute underground and circulating fluid temperatures using the flow rates within the pipes, serve as a reference for the comparison.

2. Mathematical background

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Variable</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>COP</td>
<td>Coefficient of performance of the heat pump</td>
<td>–</td>
</tr>
<tr>
<td>E1</td>
<td>Total energy demand for each time step</td>
<td>W</td>
</tr>
<tr>
<td>Ei</td>
<td>Exponential integral</td>
<td>–</td>
</tr>
<tr>
<td>EWT</td>
<td>Entering water temperature</td>
<td>K</td>
</tr>
<tr>
<td>i</td>
<td>Coordinate in the x-direction in field coordinates</td>
<td>m</td>
</tr>
<tr>
<td>j</td>
<td>Coordinate in the y-direction in field coordinates</td>
<td>m</td>
</tr>
<tr>
<td>L</td>
<td>Length of the BHE</td>
<td>m</td>
</tr>
<tr>
<td>q</td>
<td>Power demand/load</td>
<td>W</td>
</tr>
<tr>
<td>Rb</td>
<td>Thermal resistance of the borehole</td>
<td>m K W</td>
</tr>
<tr>
<td>T</td>
<td>Temperature at the BHE location</td>
<td>K</td>
</tr>
<tr>
<td>Tb</td>
<td>Temperature at the borehole wall</td>
<td>K</td>
</tr>
<tr>
<td>T0</td>
<td>Undisturbed temperature of the porous media</td>
<td>K</td>
</tr>
<tr>
<td>t</td>
<td>Time</td>
<td>s</td>
</tr>
<tr>
<td>xk</td>
<td>Coordinate of BHE k in the x-direction</td>
<td>m</td>
</tr>
<tr>
<td>yk</td>
<td>Coordinate of BHE k in the y-direction</td>
<td>m</td>
</tr>
<tr>
<td>z</td>
<td>Virtual variable</td>
<td>–</td>
</tr>
<tr>
<td>w</td>
<td>Weighting factor</td>
<td>–</td>
</tr>
<tr>
<td>λ</td>
<td>Thermal conductivity of the porous media</td>
<td>W m K</td>
</tr>
<tr>
<td>α</td>
<td>Thermal diffusivity</td>
<td>m² s⁻¹</td>
</tr>
<tr>
<td>ω</td>
<td>Response factor</td>
<td>–</td>
</tr>
<tr>
<td>k</td>
<td>Running index of BHEs</td>
<td>–</td>
</tr>
<tr>
<td>l</td>
<td>Running index of timesteps</td>
<td>–</td>
</tr>
<tr>
<td>m</td>
<td>Number of timesteps</td>
<td>–</td>
</tr>
<tr>
<td>n</td>
<td>Number of BHEs</td>
<td>–</td>
</tr>
</tbody>
</table>

2.1. Conductive heat transport in porous media (superposition principle)

A common approach for calculating two-dimensional (2D) radial temperature distribution in the underground due to the presence of a vertical BHE is the infinite line source model (Carslaw and Jaeger, 1959). Assuming an infinite homogeneous underground and a given energy transfer rate per unit length of the borehole, the temperature change (ΔT = T0 – T) caused by a single BHE with load q and with Tb as the undisturbed temperature of the underground for a certain time (t) is expressed as:

\[ΔT(Δx, Δy, t, q) = \frac{q}{4\pi L_\lambda} Ei \left(\frac{(Δx^2 + Δy^2)^{0.5}}{4\alpha t}\right)\]  

(Eq. 1)

where Ei is the so-called exponential integral, \(Δx = (i – x_k)\) and \(Δy = (j – y_k)\) are the distances to an arbitrary location \((i, j)\) with respect to a BHE centered at \((x_k, y_k)\), \(\alpha\) is the thermal diffusivity and \(\lambda\) is the bulk thermal conductivity.

With the assumption that the soil thermal properties do not depend on temperature, i.e., Eq. (1) is linear with respect to q and, due to the fact that energy is an extensive and additive variable (Hellström, 1991; Yavuzturk et al., 1999; Diao et al., 2004;
the superposition principle can be applied to Eq. (1) as:
\[
\Delta T_{i,j}(t, q_{k=1...m}) = \sum_{k=1}^{n} \Delta T_k(i - x_k, j - y_k, t, q_k)
\tag{2}
\]
where \(n\) is the number of BHEs and \(T_k\) is the temperature change at \((i,j)\) due to BHE \(k\) with an energy transfer rate \(q_k\) located at \((x_k, y_k)\).

When the workload \(q\) varies over time, it can be considered as a series of heating pulses (temporal superposition). This superposition of heat pulses has already been used in various previous works (Eskilson, 1987; Bernier et al., 2004; Marcotte and Pasquier, 2008; Michopoulos and Kyriakis, 2009; Marcotte et al., 2010). The variable energy extraction is subdivided into \(m\) time steps with constant load:
\[
\Delta T(\Delta x, \Delta y, t, q_{m=1...m}) = \sum_{l=1}^{m} \sum_{k=1}^{n} \frac{q_k - q_{k-1}}{4\pi L_k} \left( \frac{\Delta x^2 + \Delta y^2}{2\pi(t - t_l)} \right)^{0.5}
\tag{3}
\]
where \(m\) is the total number of time steps and \(q_k\) is the load during time step \(l\) which runs from \(t_{l-1}\) to \(t_l\) with \(q_0 = 0\) and \(t_0 = 0\).

By combining Eqs. (2) and (3), the temperature change at any arbitrary location in the subsurface exerted by multiple BHEs each with different time variable energy loads can be estimated through:
\[
\Delta T_{i,j}(t, q_{1...m}) = \sum_{l=1}^{m} \sum_{k=1}^{n} q_k \omega_{k,l,i,j}(i - x_k, j - y_k)
\tag{4}
\]
With:
\[
\omega_{k,l,i,j}(\Delta x, \Delta y) = \frac{1}{4\pi L_k} \left( \frac{\Delta x^2 + \Delta y^2}{2\pi(t - t_l)} \right)^{0.5} - \frac{\Delta x^2 + \Delta y^2}{4\pi(t - t_l)}
\tag{5}
\]

as the response factor of BHE \(k\) on a position \((i,j)\) within the BHE field at time step \(l\) with reference to the current time step \(t \in \{1, \ldots, m\} \) and \(l \leq t\) and \(\Delta x(i - x_k)\) and \(\Delta y(j - y_k)\).

By merging \(q_{k,l,i,j}\) and \(\omega_{k,l,i,j}\) into vectors of the form \(\vec{q} = (q_1, \ldots, q_{n,1}, \ldots, q_{n,m})\) and \(\vec{\omega}_{l,i,j} = (\omega_{1,1,i,j}, \ldots, \omega_{n,1,i,j}, \ldots, \omega_{n,n,i,j})\), Eq. (4) can be written as:
\[
\Delta T_{i,j}(t, \vec{q}) = \vec{q} \cdot \vec{\omega}(t, \vec{q})
\tag{6}
\]

where \(\Delta T_{i,j}(t, \vec{q})\) is the actual temperature change in the underground \((i,j)\) at time step \(t\) caused by the superposition of all BHEs in the field with the temporal load pattern \(\vec{q}\).

### 2.2. Optimization objectives and constraints

The efficiency of the heat pump of a geothermal installation is mainly influenced by the fluid temperature coming from the BHE field which itself hugely depends on the temperature of the underground \(T\). Therefore in the heating case, it is reasonable to keep \(\min(T)\) as high as possible to keep the maximum observed cooling of the underground \(\max(\Delta T_{i,j})\) caused by the heat extraction through the BHEs as small as possible. As a result, extreme local cooling is avoided, the temperature distribution is smoothed, and potential ecological impacts from substantial local temperature gradients are mitigated. This leads to the primary objective of the proposed load assignment optimization:

minimize \(\max(\Delta T_{i,j})\)
\tag{7}

Depending on the field thermal parameters and the given load pattern the maximum overall temperature change may not be influenced by the load assignment for certain time steps. For these cases, which are otherwise not captured by the primary objective, we minimize the maximum temperature change within each time step as a secondary objective which is given a much lower weight than the primary objective.

In order to fulfill the energy demand required from the BHE field, the sum of the loads \(q_{k,l}\) for each time step \(l\) has to equal to the required energy \(E_l\) for the given time step. This reads:
\[
E_l = \sum_{k=1}^{n} q_{k,l}, l = 1 \ldots m
\tag{8}
\]

### 2.3. Linear optimization procedure

Based on the definition of \(\Delta T_{i,j}(t, \vec{q})\) given in Eq. (6), the primary objective of minimization of the maximum overall temperature change for the entire considered time span given in Eq. (7) can be expressed as:
\[
\arg\min \left( \max(\Delta T_{i,j}(t, \vec{q})) \right) \forall (i, l, t) \in S
\tag{9}
\]

where \(\vec{q}\) is the decision variable and \(S\) is a set of all spatial and temporal reference points defined by 3-tuples \((i, l, t)\).

In a similar manner the secondary objective of minimizing the temperature change within each single time step \(l\) can be written as:
\[
\arg\min \left( \sum_{k=1}^{n} \max(\Delta T_{i,j}(l, \vec{q})) \right) \forall (i, j, t) \in S\text{ }t = l
\tag{10}
\]

In order to obtain a single objective function suitable for minimization, we combine the objective functions given in Eqs. (9) and (10) to:
\[
\arg\min \left( \sum_{l=1}^{m} \max(\Delta T_{i,j}(l, \vec{q})) + \sum_{l=1}^{m} \max(\Delta T_{i,j}(l, \vec{q})) \right) \forall (i, j, t) \in S
\tag{11}
\]

where \(w\) is a weighting factor which should be set to a large value to ensure the priority of the primary objective over the secondary objectives. \(w\) was set to 100 in this study.

Through the introduction of additional virtual variables \(z_{0...m}\) the max-norm terms of the objective function can then be rewritten as \(m + 1\) linear programs which minimize \(z_{0...m}\) and for every \(z\) reads:
\[
\min(z)
\Delta T_{i,j}(l, \vec{q}) - z_0 < 0
- \Delta T_{i,j}(l, \vec{q}) - z_0 < 0
\tag{12}
\]

with \(\vec{e}\) as the unity vector (Boyd and Vandenderghe, 2004).

Since the relationship between \(\Delta T\) and \(q\) is linear and between \(\Delta T_{i,j}\) and \(\vec{q}\) is also linear, the energy load assignment problem can be posed as a linear program:
\[
\min \left( w \cdot z_0 + \sum_{l=1}^{m} z_l \right)
\tag{13}
\]
subject to the constraints:
\[
\Delta T_{i,j}(l, \vec{q}) - z_0 < 0 \quad \forall (i, j, t) \in S
- \Delta T_{i,j}(l, \vec{q}) - z_0 < 0
\Delta T_{i,j}(l, \vec{q}) - z_l < 0 \quad \forall (i, j, l) \in S \quad l = 1 \ldots m
- \Delta T_{i,j}(l, \vec{q}) - z_l < 0
\tag{14}
\]
where \(z_{0...m}\) and \(\vec{q}\) are optimization variables.

The linear program is additionally constrained by the power demand for each time step which can be expressed as the equality constraint given in Eq. (8).

This is the monthly workload in case of a constant step size of 1 month between subsequent steps. Since in our case we only
consider heating, the load for each BHE for all time steps has to be positive. This results in the inequality constraint given by:

\[ q_{k,l} \geq 0 \quad l = 1 \ldots m, \quad k = 1 \ldots n \]  

(15)

Based on the formulation of the linear program given in Eqs. (11)–(15), the load assignment problem can be solved using standard linear program solvers such as CPLEX.

### 3. Model set up

In the present study a hypothetical BHE field scenario of 25 square lattice-like arranged BHEs with 10 m spacing \((\delta x = \delta y = 10 \text{ m})\) and 100 m length each, which is similar to the study by Katsura et al. (2009), is considered as demonstration case for optimization of individual energy extractions (Fig. 1a). The subsurface geology is approximated as homogeneous and isotropic media. It is water-saturated and in this study no groundwater flow is considered, thus only conductive heat transport is addressed. Analytical simulation is performed for an operation period of 30 years with time variable BHE-specific energy loads. The geothermal system is optimized only for heating application, and no cooling is accounted for. As common for low-enthalpy geothermal simulations, temperature dependency of hydraulic (density and viscosity of the water) as well as of thermal parameters (thermal conductivity and volumetric heat capacity) is neglected (Hecht-Méndez et al., 2010).

As shown in Fig. 1b, four reference points for each BHE \(k\) are defined. These reference points are evenly distributed on a circle with a radius of 20 cm around the BHE position \((x_k, y_k)\), which approximates the dimensions of the BHE. In order to estimate a single average subsurface temperature at the borehole wall for each BHE, we averaged \(\Delta T_{i,j,k,l}\), \(\Delta T_{i,j,k,l}\) of the corresponding four reference points at each time step \(l\). This results in effective reference points in \(S\) for each time step \(l\), respectively in \(m \times n\) reference points for the whole optimization process. Thus for the considered time span (360 months) and the given BHE field (25 BHEs), the corresponding linear program consists of 9361 optimization variables, which can be subdivided into 361 virtual variables \(z\) and 9000 actual loads \(q_{k,l}\). The number of constraints is 45,361. Assuming a subsurface composed of silts and clays, typical thermal parameters for groundwater saturated material are selected for the demonstration case (Table 1). The thermal parameters represent reference values for specific energy extraction of BHEs given by the German Engineer Association guideline for thermal use of the underground (VDI, 2001).

### Table 1

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal conductivity ((\lambda)^a)</td>
<td>W m(^{-1}) K(^{-1})</td>
<td>1.70</td>
</tr>
<tr>
<td>Thermal diffusivity ((\alpha)^a)</td>
<td>m(^2) s(^{-1})</td>
<td>7 \times 10(^{-7})</td>
</tr>
<tr>
<td>Length of the borehole ((L))</td>
<td>m</td>
<td>100</td>
</tr>
</tbody>
</table>

\(a\) Values taken from VDI (2001).

Consider a heat transfer rate of 24 W m\(^{-1}\) (VDI, 2001) and an annual runtime of 1800 h, the entire energy extraction of the BHE field is 108 MWh per year. The total energy extraction of 1 year is divided into 12 uneven intervals according to a monthly and individual heating demand typical for a GSHP system in Central Europe. Accordingly, an energy load profile for 1 year is defined, which is shown in Fig. 2. This profile approximates conditions applied in southern Germany by BHE-field designers, when no additional heat is required during the summer months. This profile is similar to the base load default profile of the EED software (version 3.15).

In the following sections, we will distinguish three cases: (i) the load optimized case, as described in the methodology section, (ii) the equal load case, which represents a non-optimized variant, assuming the same energy extraction for all BHEs, and (iii) the equal flow case, which corresponds to typical conditions of BHE fields in practice. The first two cases are simulated by the line source equation (Eq. (3)), whereas for the last case, equal volumetric flow rates of the circulating heat carrier fluid are obtained from the SBM model. The latter was applied to all (non-)optimized cases to

![Fig. 1.](image1.png)  
**Fig. 1.** (a) Top view of geothermal field with 25 BHEs used for applying the simulation-optimization procedure, (b) extended view of one BHE showing the reference points located around each BHE used for calculating a representative average borehole wall temperature.

![Fig. 2.](image2.png)  
**Fig. 2.** Extracted energy per month of a GSHP system using 25 BHEs with a specific energy extraction of 24 W m\(^{-1}\) and 1800 operating hours per year.
visualize the temperatures at 50 m depth and to compute the mixed outflow temperatures for all BHEs of the field, which enters the heat pump. SBM specific settings, which are used additionally to the values in Table 1, are listed in Table 2. These include fluid properties of water and assumed borehole resistances.

4. Results and discussion

Figs. 3–5 depict the BHE loads and the resulting temperature distribution in the subsurface for the non-optimized BHE fields with equal flow and equal loads and the load-optimized field, respectively. All three figures are divided into 16 subplots as follows: each subplot shows a combined load/temperature profile of the simulated BHE field for a defined time step \( t \) within 30 years of simulation time. The 12 subplots in the upper three rows visualize the month-wise development at half-time, that is, during the 15th operating year starting with January. The four lowermost subplots represent the temperature and load distributions for January of the 1st, 8th, 22nd and 29th years of operation and thus illustrate the long-term temperature trends of the installed BHE field. January is selected as the month with highest energy extraction and the largest temperature changes. BHEs are shown as gray shaded circles, where darker color shades denote higher BHE loads. Hence, switched-off BHEs are colored white and black BHEs are driven with the highest load possible during the actual time step. The subsurface temperature distribution (depth of 50 m) is illustrated by colors, where high absolute temperatures appear in red and lower temperatures in blue.

As anticipated, optimized and equal loading of BHEs, as well as equal flow, result in symmetric temperature anomalies that develop in the ground. Further, as illustrated by the bottom plots in Figs. 3–5, the January records of the 1st, 8th, 22nd and 29th years reveal a continuous cooling of the BHE field. Apparently,

### Table 2
Parameters specified in SBM.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fluid viscosity (15 °C)</td>
<td>kg m(^{-1}) s(^{-1})</td>
<td>(1.14 \times 10^{-3})</td>
</tr>
<tr>
<td>Fluid density</td>
<td>kg m(^{-3})</td>
<td>1000</td>
</tr>
<tr>
<td>Fluid vol. heat capacity</td>
<td>J m(^{-3}) K(^{-1})</td>
<td>(4.1912 \times 10^6)</td>
</tr>
<tr>
<td>Fluid thermal conductivity</td>
<td>W m(^{-1}) K(^{-1})</td>
<td>0.6</td>
</tr>
<tr>
<td>Borehole thermal resistance, (R_b)</td>
<td>KW m(^{-1})</td>
<td>0.0723</td>
</tr>
<tr>
<td>Borehole internal thermal resistance, (R_e)</td>
<td>KW m(^{-1})</td>
<td>0.2514</td>
</tr>
</tbody>
</table>
the regeneration time during the summer period is too short to balance the deficit by conductive lateral heat supply in all three cases, which is a well-known long-term behavior of such GSHP systems (Eugster and Rybach, 2000). This is also reflected by the small changes that can be observed during the small or no energy extraction phase in the summer season between May and August.

Figs. 3-5 reveal that long-term BHE operations can generate substantially decreased temperatures that locally fall below 2.5 °C. The relatively high observed temperature changes, \( \Delta T \), correspond to the results of recent investigations, for instance by Priarone et al. (2009), who obtained even higher values for heating dominated cases with large BHE fields. Rybach and Eugster (2010) observed similar values close to the borehole wall of a single BHE application at the Elgg site in Switzerland. Since the relationship between \( \Delta T \) and the loads \( \vec{q} \) is linear, \( \Delta T \) can easily be limited to a desired value of \( \Delta T \) by scaling the \( \vec{q} \) obtained from the optimizer with the ratio between the desired \( \Delta T \) and the actual \( \Delta T \). In other words, the relative difference between optimized and non-optimized BHE fields remains the same even if much smaller total specific heat extraction rates are assumed and/or a stricter \( \Delta T \) constraint has to be obeyed.

At first sight, the differences between the three cases do not appear significant. While equal flows and workloads tend to generate local anomalies in the BHE field, the optimized system yields more smoothed temperature contours. The temperature anomalies successively develop for all operation modes, but are most concentrated in the equal load case. This reflects that, in particular, the interior BHEs in the field are insulated by the outer BHEs, which averts sufficient conductive energy supply from the ambient ground. Even in the equal flow case, which represents a self-regulated system, where heat transfer is automatically smaller in low-temperature regions, a similar trend can be observed. Central BHEs cool the ground much more than the outer BHEs. Still, this case, which represents standard practice, performs better than the more hypothetical equal load case: the low temperature extremes are less pronounced after the full operation period of 30 years (Figs. 3-5).

Figs. 3 and 5 show that the load distribution does not exhibit equal extraction rates for all BHEs during equal flow and optimized load operation modes. In fact, the individual BHE loads crucially differ depending on the position of the individual BHE within the field and at a certain time-step. While for equal flow this is self-regulated according to the temperature gradients at each BHE, load optimization yields a system-specific strategy. It turns out that it is beneficial to deactivate the BHEs with the largest influence on neighboring BHEs, starting at the months with lowest energy demand, and to reactivate them when the energy demand rises again. Thus, the global optimum for the used load profile suggests deactivating first...
Fig. 5. Temperature distribution and BHE workloads for the optimized case. Each circle represents a BHE with its corresponding load in grayscale. Darker color shades denote higher BHE loads. The subsurface temperature distribution at a depth of 50 m is illustrated by colors, where high absolute temperatures appear in red and lower temperatures in blue. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of the article.)

Fig. 6. Temperature differences in the subsurface between the optimized and the equal flow case at the end of the heating period (March) of the final year (30) of the simulated time frame. (a) Shows the areal temperature difference at 50 m depth between both cases. (b) Depicts a cross section of the temperature change from one corner of the BHE field to the opposed one (dashed line in a).
Additionally, a cross section of the temperature change along the areal temperature difference in 50 m depth between both cases. This is illustrated as the higher lateral cooling of the ground is achieved. This is illustrated more detail in Fig. 6, which inspects the underground temperature change after the heating period in the final year. Fig. 6a shows the areal temperature difference in 50 m depth between both cases. Additionally, a cross section of the temperature change along the dashed line is depicted in Fig. 6b. The differences in Fig. 6a reach up to 1.25 °C (i.e., 18%), which represents the strongest cooling at the central BHE. By load optimization, the lower central heat extraction is compensated by relatively higher loads at the BHE field boundary. This is reflected by the areas (dark blue) at the fringe with negative temperature differences, and shown in Fig. 6b when comparing the temperatures of the BHEs at a distance of 10 m and then 50 m from the corner of the field. Another aspect that can be extracted from the plots in Fig. 6 is that even if the optimized system does not manage to generate equal temperatures, the variability is much less pronounced. The positive differences in the center of the field demarcate relative higher cooling by the equal flow case when compared to the optimized case. These are considerably more severe than the negative differences which are located at the fringe of the field.

Of special interest is the temperature that arrives at the heat pump, which can also be used to compare the three different operation modes. The lower this temperature is, the smaller the expected seasonal performance factor and thus the worse the efficiency of the entire geothermal heating system. Fig. 7 reports the mixed outflow temperature evolution of the working fluid of all BHEs of the non-optimized equal flow case and the load optimized case in the field. These are obtained by averaging the monthly BHE-specific outflow temperatures simulated by SBM. Similar to the findings from previous studies (e.g., Signorelli et al., 2004), sinusoidal oscillation reflects alternating use phases and regeneration phases. Since the subsurface is not able to regenerate during the course of the year, annual mean temperatures decline with progressing use. Due to averaging, the differences between the two cases are only very small. Obviously, mixing rules out the extremes as observed when comparing the in situ temperature distribution (Figs. 3–6). Even so, as highlighted in the zoomed section in Fig. 7, the optimized load case yields slightly higher temperatures than the alternative cases. This demonstrates that even if loads are used as proxy for the combined simulation-optimization, better strategies can be found than standard practice. Since the difference is rather small, however, in practice slight performance improvement potential may not be sufficient for justifying the application of additional control systems. In fact, the advantage of an optimized system is the possibility to extract a higher amount of energy while complying with given temperature thresholds or environmental constraints.

5. Conclusions

A new approach for the optimization of scheduled energy extraction of closed shallow geothermal systems based on linear programming is presented. In this work, flow regulation is not implicitly addressed in the mathematical formulation of the optimization problem, thus BHE workloads will be optimized as a surrogate for flow rates. Combined spatial and temporal superposition of the infinite line source analytical solution is used to simulate temperature changes in the underground caused by multiple adjacent borehole heat exchangers (BHEs), which operate with variable energy loads. By exploiting the linear relationship between the individual loads of the BHEs and the temperature changes in the underground we are able to formulate a linear optimization problem for assigning globally optimal loads for each BHE for each time step. The optimization objective is to minimize the ground temperature changes and therefore maximize heat pump performance, comply with regulatory thresholds for induced temperature changes and mitigate environmental impacts. To determine if the workload optimization allows for improvement of BHE field design even without considering flow regulation, the optimized solutions obtained by this proxy were compared to a BHE field implementation, which operates at equal flows and loads. Except for the optimization step, all simulations were accomplished by the SBM code (Eskilson, 1986; Pahud et al., 1996).

The procedure is demonstrated for a simplified hypothetical case with 25 BHEs that are operated for 30 years to accomplish seasonally variable heating energy requirements typical for larger office buildings, schools or district heating systems. In the current study only heat conduction, i.e., no significant groundwater flow is considered in a homogeneous ground. The optimized BHE fields result in a mitigation of long-term temperature decrease and local temperature anomalies, and a more balanced, more lateral cooling of the ground is achieved. Current practical applications using equal flow rates for all BHEs achieve acceptable mixed outflow temperatures, but the absolute temperature decrease in the subsurface can additionally be diminished up to 18% if optimization is incorporated. This instance allows for significantly higher energy extraction rates without increasing environmental impacts and respectively a better compliance with statutory provisions. Since the developed optimization method does not distinguish between any assumptions regarding the geometry of the field and the geological, hydrogeological and thermal parameters of the ground, it can be easily transferred to other cases and be readily implemented in planning software. Depending on the specific problem, the selected monthly resolution of the load profile can also be replaced by a coarser or finer resolution. As an extension, cooling can be accounted for. Almost any given BHE field geometry can be inspected, if the underlying superposition assumptions hold. However, since the mutual impact factors of the BHEs in the field are calculated using the infinite line source model, the presented method can currently only be used for conductive cases without groundwater flow. Alternative BHE simulation would therefore be necessary, if advective heat transport in the ground is significant. Likewise, the optimization procedure can also be extended for additional analytical solutions, as for instance the finite line
source model (Zeng et al., 2002) or finite moving line source model (Molina-Giraldo et al., 2011).

Large matrices are utilized to represent the mutual influence coefficients between the BHEs for all time steps. Implemented in a computer, they easily grow to several gigabytes in size. This sets an upper limit on the number of BHEs and of time steps depending on the platform used. The computational time was approximately 30 min for the calculation of the coefficients at and 3 h for solving the linear program on a 3.6 GHz quadcore CPU utilizing 6 GB of RAM. In view of the growing computational power, however this is not considered to be a severe shortcoming in the future. As a remedy, it is often desirable to search for close-optimal solutions that can be obtained by separation of the full operation problem into sub-problems, e.g., by successively optimized separate operation intervals.

Acknowledgements

This work was supported by the Deutsche Bundesstiftung Umwelt (DBU), the EU FP7 ECO-GHP project, and the Federal Ministry for Education and Research (BMBF) scholarship program for International Postgraduate Studies in Water Technologies (IPSWaT). We thank David Kuntz for providing the load profile used in this study. We would also like to thank Daniel Pahud for providing the SBM model and the reviewers who helped to considerably improve the paper with their constructive feedback.

References